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The reggeon $\to$ 2reggeons + particle vertex in the Lipatov effective action formalism

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Abstract. The vertex for gluon emission during the splitting of a reggeized gluon into two is constructed in the framework of the Lipatov effective action formalism. Its reduction to a pure transverse form for the diffractive amplitude gives the standard Bartels vertex plus an additional contribution, corresponding to the emission from a pointlike splitting vertex. This additional contribution turns out to be given by a longitudinal integral divergent both in the ultraviolet and infrared. A certain specific recipe for this part, including the principal value prescription for the integration, allows one to eliminate this unwanted contribution.

1 Introduction

Particle interaction in Regge kinematics, when the energy $\sqrt{s}$ is large and much greater than the typical transferred momentum $\sqrt{-t}$, in perturbative QCD is described by the exchange of reggeized gluons ('reggeons') accompanied by the emission of real gluons ('particles'). In the high-color approximation this picture reduces to the exchange of BFKL pomerons [1–3] that fuse and split with known triple-pomeron vertices [4–6]. A summation of the resulting diagrams in the tree approximation leads to the BK equation for deep inelastic scatering [7–10] or to a pair of equations for in- and outgoing pomerons for nucleus–nucleus scattering [11]. The contribution from loop diagrams may be studied in the formalism of an effective field theory [12] or in the competing Hamiltonian formalism in terms of the gluon variables (see e.g. [13] and references therein).

In all these developments the basic building blocks have been calculated directly from the relevant 4-dimensional Feynman diagrams in Regge kinematics and converted into the corresponding expressions in the 2-dimensional transverse space (momentum or coordinate). This procedure is easily realized for the simplest elements, such as particle emission during the exchange of a single reggeized gluon, but it becomes very cumbersome for more complicated processes. A potentially powerful formalism of an effective action has been proposed by Lipatov [14], in which the longitudinal and transverse variables are separated from the start and one arrives at a theory of interaction of reggeized gluons and particles described by independent fields. This formalism, in principle, allows one to automatically calculate all diagrams in the Regge kinematics in a systematic and self-consistent way. However, the resulting vertices are 4-dimensional and reduction of them to the 2-dimensional transverse form still has to be done.

Up to the present several applications of this formalism have been made, and a number of interaction vertices have been calculated. They include vertices for the transition of a reggeon into several particles or into a reggeon and several particles [15]. In this paper we study the vertex for the transition of a reggeon into a pair of reggeons and a particle (a $RRRP$ vertex in the terminology of [15]) in the effective action formalism. The 2-dimensional form of this vertex (the 'Bartels' vertex) is well known [4, 16] (see also [17] for its form in the coordinate space). The 4-dimensional vertex found resembles the Bartels vertex, although it contains a new structure absent in the latter and of course longitudinal variables. Unlike the vertices obtained so far in the effective action formalism, reduction of the $RRRP$ vertex to the 2-dimensional form involves a non-trivial integration in the loop formed by the two reggeons and the target. We discuss this integration and show that for a diffractive diagram, in which both the projectile and target are just quarks, a literal integration over the longitudinal variables is impossible (divergent). The Bartels vertex is obtained only if certain ad hoc rules are followed, which reduce to neglecting all small longitudinal momenta in the target factor and to a subsequent integration over the minus component of the loop momentum according to the principal value prescription.

2 Feynman rules from the effective action

The Feynman rules from the effective action were presented in [15] and we only recapitulate them here for convenience and to fix our notation. We use the light-cone variables defined by the light-cone unit vectors $n^{\pm}_{\mu} =$

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$(1,0,0,\mp 1)$. So $a_\pm = a_\mu n^\pm_\mu = a_0 \pm a_3$ and $ab = a_\mu b_\mu = (1/2)(a_+ b_- + a_- b_+) + a_\perp b_\perp$. The metric tensor is $g_{+-} = g_{-+} = 1/2$, $g_{11} = g_{22} = -1$. In a standard way we denote $D_\mu = \partial_\mu + gV_\mu$, where $V_\mu = -\mathrm{i}V^a_\mu T^a$ is the gluon field, and the T^a are the SU(N) generators in the adjoint representation. The reggeon field $A_\pm = -\mathrm{i}A^a_\pm T^a$ satisfies the kinematical condition

$$\partial_+ A_- = \partial_- A_+ = 0\,. \quad (1)$$

The field A_+ comes from the region with a higher rapidity, its momentum q_- is small; the field A_- comes from the region with a smaller rapidity, and its q_+ is small. The QCD Lagrangian for the particle (gluon) field V is standard, and so are the Feynman rules. Parts of the effective action describing the reggeon field A are presented in [14], to which we refer as L. The effective action is (L.210),

$$\begin{aligned}\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}}(V_\mu + A_\mu) \\ + \mathrm{Tr}\big((\mathcal{A}_+(V_+ + A_+) - A_+)\partial^2_\perp A_- \\ + (\mathcal{A}_-(V_- + A_-) - A_-)\partial^2_\perp A_+\big)\,, \end{aligned} \quad (2)$$

where

$$\begin{aligned}\mathcal{A}_\pm(V_\pm) &= -\frac{1}{g}\partial_\pm \frac{1}{D_\pm}\partial_\pm * 1 = \sum_{n=0}(-g)^n V_\pm \left(\partial^{-1}_\pm V_\pm\right)^n \\ &= V_\pm - gV_\pm \partial^{-1}_\pm V_\pm + g^2 V_\pm \partial^{-1}_\pm V_\pm \partial^{-1}_\pm V_\pm + - \ldots \end{aligned} \quad (3)$$

The shift $V_\mu \to V_\mu + A_\mu$ with $A_\perp = 0$ is to exclude the direct transitions $V \leftrightarrow A$. The propagator is determined by terms quadratic in the fields. In the Feynman gauge $\mathcal{L}^{\text{quadr}}_{\text{QCD}} = \mathrm{Tr}(-V_\mu \partial^2 V_\mu)$, which after the substitution $V_\pm \to V_\pm + A_\pm$ leads to

$$\begin{aligned}&\mathrm{Tr}\big(-(V_\nu + A_\nu)\partial^2(V_\nu + A_\nu)\big) \\ &= \mathrm{Tr}\big(-V_\nu\partial^2 V_\nu\big) + \mathrm{Tr}\big(-V_+\partial^2_\perp A_- - V_-\partial^2_\perp A_+\big) \\ &\quad + \mathrm{Tr}\big(-A_-\partial^2_\perp A_+\big)\,. \end{aligned} \quad (4)$$

The second term cancels with a similar term in the induced action, and the quadratic terms that remain are

$$\begin{aligned}&\mathrm{Tr}\big(-V_\nu\partial^2 V_\nu\big) + \mathrm{Tr}\big(-A_-\partial^2_\perp A_+\big) \\ &= \frac{1}{2}V^a_\nu \partial^2 V^a_\nu + \frac{1}{4}\big(A^a_-\partial^2_\perp A^a_+ + A^a_+\partial^2_\perp A^a_-\big)\,. \end{aligned} \quad (5)$$

Thus, the reggeon propagator in momentum space is

$$\langle A^a_+ A^b_- \rangle = -\mathrm{i}\frac{2\delta_{ab}}{q^2_\perp}\,. \quad (6)$$

The reggeon → reggeon + particle (‘Lipatov’) vertex (Fig. 1, left) is

$$\frac{gf^{ab_2d}}{2}\left[q_\sigma + q_{2\sigma} + \left(\frac{q_2^2}{q_+} - q_{2-}\right)n^+_\sigma + \left(\frac{q^2}{q_{2-}} - q_+\right)n^-_\sigma\right]. \quad (7)$$

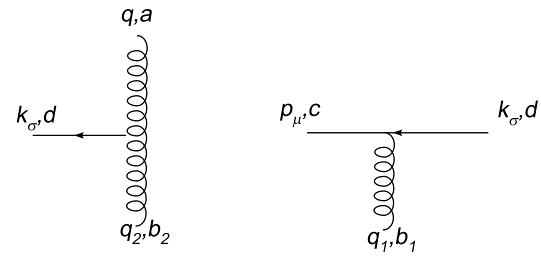

Fig. 1. Vertices for reggeon → reggeon + particle (*left*) and particle → reggeon + particle (*right*)

Note that in the gauge $V_+ = 0$ (for a $p_A + p_B$ collision it is equivalent to $p_B V = 0$) the polarization vector has the form

$$\epsilon_\mu(k) = \epsilon^\perp_\mu(k) - \frac{k\epsilon^\perp}{k_+}n^+_\mu\,. \quad (8)$$

Convoluting the vertex (7) with this polarization vector, we find

$$gf^{ab_2d}q^2_\perp\left(\frac{q\epsilon_\perp}{q^2_\perp} - \frac{k\epsilon_\perp}{k^2_\perp}\right). \quad (9)$$

This gives the form of the Lipatov vertex widely used in the literature.

Other ingredients of the technique will be derived in the course of the derivation. They use parts of the effective action as listed below. The part containing two fields A_- is (L.276),

$$\begin{aligned}\mathcal{L}_{A_-A_-} &= -\frac{1}{4}\mathrm{Tr}([D_+, A_-])^2 \\ &= -\frac{1}{4}\mathrm{Tr}\big([\partial_+ - \mathrm{i}gV^a_+T^a, -\mathrm{i}A^b_-T^b]\big)^2 \\ &= -\frac{1}{4}\mathrm{Tr}\big(\partial_+A^b_-(-\mathrm{i}T^b) + gV^a_+A^b_-(-\mathrm{i}f^{abc}T^c)\big)^2. \end{aligned} \quad (10)$$

The part containing one field A_- and one A_+ is (also (L.276))

$$\begin{aligned}\mathcal{L}_{AAV} = g\mathrm{Tr}\Big(&[V^\perp_\sigma, A_+]\partial^\perp_\sigma A_- + [V^\perp_\sigma, A_-]\partial^\perp_\sigma A_+ \\ &+ \frac{1}{2}[A_-, A_+]\partial_+V_- + \frac{1}{2}[A_+, A_-]\partial_-V_+\Big) \\ &+ g\mathrm{Tr}\big(-[V_+, \partial^{-1}_+ A_+]\partial^2_\perp A_- \\ &\quad - [V_-, \partial^{-1}_- A_-]\partial^2_\perp A_+\big)\,. \end{aligned} \quad (11)$$

Here the first big bracket corresponds to the standard triple vertex and the second one to the induced one. The part of the action containing two fields A_-, one A_+ and V_μ is (L.277),

$$\begin{aligned}\mathcal{L}_{AAAV} = -\frac{g}{2}\mathrm{Tr}\Big(&-[D_+, A_-][A_-, A_+] \\ &+ 2\partial_-\frac{1}{D_-}A_-\frac{1}{D_-}A_-\frac{1}{D_-}\partial_-\partial^2_\perp A_+\Big)\,. \end{aligned} \quad (12)$$

Here

$$\frac{1}{D_-}=\frac{1}{\partial_-+gV_-}=\partial_-^{-1}\frac{1}{1+g\partial_-^{-1}V_-}$$
$$=\partial_-^{-1}\sum_{n=0}\left(-g\partial_-^{-1}V_-\right)^n=\partial_-^{-1}-g\partial_-^{-1}V_-\partial_-^{-1}+\dots \tag{13}$$

Finally, the part of the action containing two gluon and one reggeon field is (L.275),

$$\mathcal{L}_{AVV}=g\mathrm{Tr}\big[\big([V_\nu,\partial_+V_\nu]-[\partial_\nu V_\nu,V_+]-2[V_\nu,\partial_\nu V_+]$$
$$-V_+\partial_+^{-1}V_+\partial_\perp^2\big)A_-\big]\,. \tag{14}$$

The three first terms give the standard vertex:

$$-\frac{g}{2}f^{b_1cd}\big((k+p)_+g_{\mu\sigma}+(p-2k)_\mu n_\sigma^+ +(k-2p)_\sigma n_\mu^+\big)\,, \tag{15}$$

and the last term defines the induced vertex:

$$gf^{b_1cd}\frac{q_1^2}{2p_+}n_\mu^+n_\sigma^+\,. \tag{16}$$

3 Calculation of the diagrams

The total contribution to the vertex reggeon → two reggeons + particle is represented by the four diagrams shown in Fig. 2, in which particles are shown by solid lines and reggeons by wavy lines.

3.1 Diagram 3 of Fig. 2

We start from the simplest diagram 3 of Fig. 2. For this diagram we have to know the vertex $\langle VA_-A_-\rangle$. The Lagrangian term with two fields A_- is given by (10). The standard triple vertex is obtained as a product of the first and second terms in the brackets after taking the square. However, it vanishes, since the first term in the brackets is zero. It is important here that the expansion of the P-exponential in the effective action formalism generates only an induced vertex $\langle VA_-A_+\rangle$ but not $\langle VA_-A_-\rangle$.

Thus diagram 3 of Fig. 2 does not give any contribution in our kinematics.

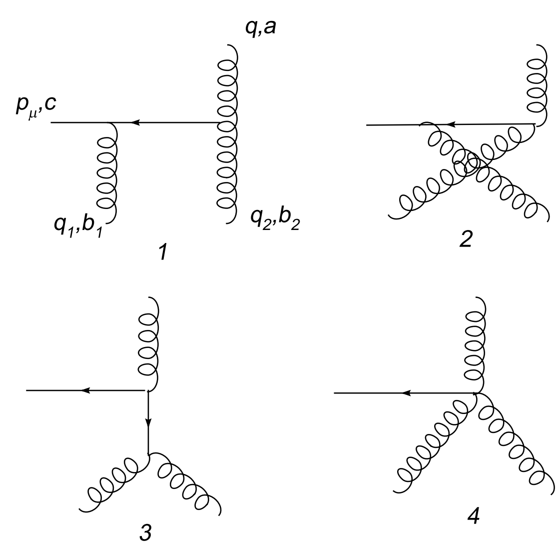

Fig. 2. Reggeon diagrams for the vertex reggeon → 2 reggeons +particle

3.2 Diagram 4 of Fig. 2

For diagram 4 of Fig. 2 we have to know $\langle A_+A_-A_-V_\mu\rangle$. It is contained in the part of the Lagrangian (12). In it the factor $1/D_-$ is a remnant of the P-exponential:

$$U(V_-)=P\exp\left(-\frac{g}{2}\int_{-\infty}^{x_+}\mathrm{d}x'_+V_-(x'_+)\right)\,, \tag{17}$$

so that

$$\partial_-U(V_-)=2\frac{\partial}{\partial x_+}U(V_-)=-gV_-U(V_-)\,. \tag{18}$$

Taking into account that $U_{g=0}=1$, we get

$$U(V_-)=\sum_{n=0}\left(-g\partial_-^{-1}V_-\right)^n=\frac{1}{D_-}\partial_-\,. \tag{19}$$

The terms of $\mathcal{L}_{AAAV}$ that determine the vertex with only one gluon are

$$-\frac{g}{2}\mathrm{Tr}\,(-g[V_+,A_-][A_-,A_+]$$
$$-2g\left(V_-\frac{1}{\partial_-}A_-\frac{1}{\partial_-}A_-\partial_\perp^2A_+\right.$$
$$+A_-\frac{1}{\partial_-}V_-\frac{1}{\partial_-}A_-\partial_\perp^2A_+$$
$$\left.\left.+A_-\frac{1}{\partial_-}A_-\frac{1}{\partial_-}V_-\partial_\perp^2A_+\right)\right)\,. \tag{20}$$

The first term is the standard quartic interaction; the other ones give the induced interaction.

We pass to the momentum representation: for the incoming momentum $\mathrm{i}\partial_\mu\to p_\mu$ and for the outgoing momentum $\mathrm{i}\partial_\mu\to -p_\mu$. We find accordingly

$$\frac{1}{\partial_-}A_-(q_1)\to\frac{\mathrm{i}}{-q_{1-}}\,,\quad \frac{1}{\partial_-}V_-(p)\to\frac{\mathrm{i}}{-p_-}\,,$$
$$\partial^2A_+(q)\to -q_\perp^2\simeq -q^2. \tag{21}$$

The last identity follows from $\partial^2A_+=(\partial_+\partial_-+\partial_\perp^2)A_+=\partial_\perp^2A_+$. If the derivative acts on a product of fields, $\partial_-(A_1A_2\dots A_n)$, then it corresponds to the sum of momenta in the momentum representation. This is also true for the inverse operator $1/\partial_-$: it corresponds to the sum of the momenta in the denominator.

Note the kinematical relations among the momenta:

$$p=q-q_1-q_2\,,q_-\ll p_-\sim q_{1-}\sim q_{2-}\,,$$
$$q_{1+}\sim q_{2+}\ll p_+=q_+\,,p_-=-q_{1-}-q_{2-}\,,$$
$$q_\perp\sim q_{1\perp}\sim q_{2\perp}\sim p_\perp\,. \tag{22}$$

Each vertex bears a factor i, and the diagram as a whole has a factor 1/i for the amplitude, which is omitted.

We find for the induced vertex the following. The common factor is $g^2\mathrm{i}(-\mathrm{i})^4(-\mathrm{i})^2 n^-_\mu$ (where $(-\mathrm{i})$ is from each of the fields, $(-\mathrm{i})^2$ from two $1/\partial_-$, and n^-_μ from V_-). We use $q_- = 0$, so that $1/\partial_-$ does not act on A_+). We have

$$\begin{aligned} V_4^{\mathrm{ind}} &= -\mathrm{i}g^2 n^-_\mu(-q_\perp^2) \\ &\quad\times \mathrm{Tr}\bigg(\frac{T^cT^{b_1}T^{b_2}T^a}{(q_{1-}+q_{2-})q_{2-}} + \frac{T^{b_1}T^cT^{b_2}T^a}{(p_-+q_{2-})q_{2-}} \\ &\qquad + \frac{T^{b_1}T^{b_2}T^cT^a}{(p_-+q_{2-})p_-}\bigg) \\ &\quad + (q_1 \leftrightarrow q_2, b_1 \leftrightarrow b_2) \\ &= -\mathrm{i}g^2 n^-_\mu q_\perp^2 \\ &\quad\times \mathrm{Tr}\bigg(\frac{1}{p_-q_{2-}}(T^cT^{b_1}T^{b_2}T^a + T^{b_2}T^{b_1}T^cT^a) \\ &\qquad + \frac{1}{p_-q_{1-}}(T^{b_1}T^{b_2}T^cT^a + T^cT^{b_2}T^{b_1}T^a) \\ &\qquad + \frac{1}{q_{1-}q_{2-}}(T^{b_1}T^cT^{b_2}T^a + T^{b_2}T^cT^{b_1}T^a)\bigg) . \end{aligned} \tag{23}$$

We have used $q_{1-} + q_{2-} = -p_-$. Also using

$$\frac{1}{p_-q_{1-}} + \frac{1}{p_-q_{2-}} = -\frac{1}{q_{1-}q_{2-}}$$

and the cyclic symmetry under the trace, we find

$$\begin{aligned} V_4^{\mathrm{ind}} &= -\mathrm{i}g^2 n^-_\mu q_\perp^2 \\ &\quad\times \mathrm{Tr}\bigg(\frac{1}{p_-q_{2-}}([T^cT^{b_1}]T^{b_2}T^a + T^{b_2}[T^{b_1}T^c]T^a) \\ &\qquad + \frac{1}{p_-q_{1-}}(T^{b_1}[T^{b_2}T^c]T^a + [T^cT^{b_2}]T^{b_1}T^a)\bigg) \\ &= \mathrm{i}\frac{g^2}{2} n^-_\mu q_\perp^2 \left(\frac{1}{p_-q_{2-}} f^{b_1cd}f^{ab_2d} + \frac{1}{p_-q_{1-}} f^{ab_1d}f^{b_2cd}\right) . \end{aligned} \tag{24}$$

To find the contribution from the quartic vertex, we only have to introduce the indices ± into the general expression and use the light-cone values of $g_{\mu\nu}$. We get (in the same manner as from the effective action)

$$V_4 = -\mathrm{i}\frac{g^2}{4} n^+_\mu (f^{b_1cd}f^{ab_2d} + f^{ab_1d}f^{b_2cd}) . \tag{25}$$

So the total contribution from diagram 4 of Fig. 2 is

$$\begin{aligned} V_4^{\mathrm{tot}} = \frac{\mathrm{i}g^2}{4}\bigg[& f^{b_1cd}f^{ab_2d}\left(2\frac{q_\perp^2 n^-_\mu}{p_-q_{2-}} - n^+_\mu\right) \\ & + f^{ab_1d}f^{b_2cd}\left(2\frac{q_\perp^2 n^-_\mu}{p_-q_{1-}} - n^+_\mu\right)\bigg] . \end{aligned} \tag{26}$$

In the gauge $V_+ = 0$ the convolution of (26) with the polarization vector (8) gives

$$\mathrm{i}g^2\frac{p\epsilon_\perp}{p_\perp^2} q_\perp^2 \left(\frac{1}{q_{2-}} f^{b_1cd}f^{ab_2d} + \frac{1}{q_{1-}} f^{b_1ad}f^{cb_2d}\right) . \tag{27}$$

Note that only the induced vertex contribution remains; the contribution from the standard quartic vertex vanishes.

4 Diagrams 1 and 2 of Fig. 2

We pass to diagrams 1 and 2 as shown in Fig. 2. They only differ by a permutation of the reggeons A_-, that is, $q_1 \leftrightarrow q_2$, $b_1 \leftrightarrow b_2$. So we study only the first one. To write the expression we have to know the vertices of reggeon → reggeon + gluon and reggeon–gluon–gluon.

The first (Lipatov) vertex is known and is given by (7). It is instructive to see how it is derived from (11). In the momentum representation $A_+ \to (-\mathrm{i}q, a)$, $A_- \to (\mathrm{i}q_2, b)$ and $V \to (\mathrm{i}k, d, \sigma)$. Factors originate as follows: $(-\mathrm{i})^3$ is from the fields, $(-\mathrm{i})$ from the derivatives and $\mathrm{i}/2$ from the commutator and trace. We get for the standard vertex

$$\begin{aligned} & \frac{(-\mathrm{i})^4\mathrm{i}g\mathrm{i}}{2}\bigg(-q^\perp_{2\sigma}f^{dab_2} + q^\perp_{2\sigma}f^{db_2a} \\ &\quad + \frac{1}{2}n^-_\sigma(-k_+)f^{b_2ad} + \frac{1}{2}n^+_\sigma(-k_-)f^{ab_2d}\bigg) \\ &= \frac{g}{2}f^{ab_2d}\left(q^\perp_{2\sigma} + q^\perp_\sigma - \frac{1}{2}n^-_\sigma k_+ + \frac{1}{2}n^+_\sigma k_-\right) \\ &= \frac{g}{2}f^{ab_2d}\left(q^\perp_{2\sigma} + q^\perp_\sigma - \frac{1}{2}n^-_\sigma q_+ - \frac{1}{2}n^+_\sigma q_{2-}\right) \\ &= \frac{g}{2}f^{ab_2d}\left((q+q_2)_\sigma - n^-_\sigma q_+ - n^+_\sigma q_{2-}\right) . \end{aligned} \tag{28}$$

This coincides with the expression for the standard triple vertex $gf^{db_2a}\gamma_{\sigma+-}(k, q_2)$. Now for the induced vertex ($\partial_+^{-1}A_+ \to \mathrm{i}/q_+$, $\partial_-^{-1}A_- \to -\mathrm{i}/q_{2-}$):

$$\begin{aligned} & -\frac{1}{2}(-\mathrm{i})^4\mathrm{i}g\mathrm{i}\bigg(-\frac{1}{q_+}n^+_\sigma f^{dab_2}(-q_{2\perp}^2) \\ &\quad + \frac{1}{q_{2-}}n^-_\sigma f^{db_2a}(-q_{2\perp}^2)\bigg) \\ &= \frac{1}{2}gf^{ab_2d}\left(\frac{q_2^2}{q_+}n^+_\sigma + \frac{q^2}{q_{2-}}n^-_\sigma\right) . \end{aligned} \tag{29}$$

The full vertex is then given by (7). It has to be transversal. Indeed, convolution with $k_\sigma = (q - q_2)_\sigma$ gives

$$\begin{aligned} & (q-q_2)_\sigma(q+q_2)_\sigma + \left(\frac{q_2^2}{q_+} - q_{2-}\right)q_+ + \left(\frac{q^2}{q_{2-}} - q_+\right)(-q_{2-}) \\ &= q^2 - q_2^2 + q_2^2 - q^2 - q_{2-}q_+ + q_+q_{2-} \\ &= 0 . \end{aligned} \tag{30}$$

Further simplification is possible in the gauge $V_+ = 0$: then terms with n^+_μ go and the vertex acquires the form (9).

To calculate diagram 1 of Fig. 2 we finally need the vertex gluon → reggeon + gluon (Fig. 1, right). The relevant terms in the effective action are (14). Using the kinematical relation $k = p + q_1$, which leads to $p - 2k = -p - 2q_1$, $k - 2p = q_1 - p$ and $k_+ = p_+$, we find the total vertex from (15) and (16) to be

$$\frac{gf^{b_1cd}}{2}\Big(-2p_+g_{\mu\sigma}+(p+2q_1)_\mu n^+_\sigma +(p-q_1)_\sigma n^+_\mu+\frac{q_1^2}{p_+}n^+_\mu n^+_\sigma\Big)\,. \tag{31}$$

Diagram 1 of Fig. 2 is obtained by convoluting this vertex with the vertex AAV and the gluon propagator $-\mathrm{i}g_{\sigma\sigma'}\delta_{dd'}/k^2$. We find

$$\begin{aligned}
&\frac{-\mathrm{i}g^2f^{b_1cd}f^{ab_2d}}{4k^2}\\
&\times\Big(-2p_+g_{\mu\sigma}+(p+2q_1)_\mu n^+_\sigma+(p-q_1)_\sigma n^+_\mu+\frac{q_1^2}{p_+}n^+_\mu n^+_\sigma\Big)\\
&\times\Big[q_\sigma+q_{2\sigma}+\Big(\frac{q_2^2}{q_+}-q_{2-}\Big)n^+_\sigma+\Big(\frac{q^2}{q_{2-}}-q_+\Big)n^-_\sigma\Big]\\
&=\frac{-\mathrm{i}g^2f^{b_1cd}f^{ab_2d}}{4k^2}\\
&\times\big[q_+(-2q-2q_2+p+2q_1)_\mu+((q+q_2)(p-q_1)+q_1^2)n^+_\mu\\
&+\Big(\frac{q_2^2}{q_+}-q_{2-}\Big)(-q_+)n^+_\mu+\Big(\frac{q^2}{q_{2-}}-q_+\Big)\\
&\times\Big(-2q_+n^-_\mu+2(p+2q_1)_\mu+(p_--q_{1-}+2\frac{q_1^2}{q_+})n^+_\mu\Big)\Big]\,.
\end{aligned} \tag{32}$$

Diagram 2 of Fig. 2 is obtained by the interchange of reggeons 1 and 2.

4.1 Total vertex

The total vertex is given by the sum of diagrams 1, 2 and 4 of Fig. 2 (since the contribution of diagram 3 of Fig. 2 is zero) The sum of diagrams 1, 2 and 4 of Fig. 2 is found to be transversal (convolution with p_μ is zero) as it should.

To find the transition amplitude we have to convolute the sum of diagrams 1, 2 and 4 of Fig. 2 with the polarization vector $\epsilon^*_\mu(p)$ and divide by i. In the gauge $V_+=0$, we have

$$\epsilon_\mu(p)=\epsilon^\perp_\mu-\frac{(p\epsilon_\perp)}{p^+}n^+_\mu\,. \tag{33}$$

It satisfies

$$\epsilon p=0\,,\quad \epsilon_+=0\,,\quad \epsilon_-=-\frac{2(p\epsilon_\perp)}{p_+} \tag{34}$$

on the mass shell $p^2=0$. The result of the convolution is

$$\begin{aligned}
&g^2\frac{f^{b_1cd}f^{ab_2d}}{(q-q_2)^2}\\
&\times\Big[q_+(q\epsilon^*_\perp)+\frac{q^2}{q_{2-}}\Big(-(q-q_2)\epsilon^*_\perp+\frac{(q-q_2)^2}{p_\perp^2}(p\epsilon^*_\perp)\Big)\Big]\\
&+g^2\frac{f^{b_2cd}f^{ab_1d}}{(q-q_1)^2}\\
&\times\Big[q_+(q\epsilon^*_\perp)+\frac{q^2}{q_{1-}}\Big(-(q-q_1)\epsilon^*_\perp+\frac{(q-q_1)^2}{p_\perp^2}(p\epsilon^*_\perp)\Big)\Big]\,.
\end{aligned} \tag{35}$$

If we denote

$$\begin{aligned}
u_1&=q-q_1\,, & u_2&=q-q_2\,,\\
F_1&=f^{b_2cd}f^{ab_1d}, & F_2&=f^{b_1cd}f^{ab_2d},
\end{aligned} \tag{36}$$

then (35) can be rewritten as

$$\begin{aligned}
&g^2F_1\Big(\frac{q_+}{u_1^2}(q\epsilon^*_\perp)+\frac{q^2}{u_1^2u_{1-}}(u_1\epsilon^*_\perp)-\frac{q^2}{u_{1-}p_\perp^2}(p\epsilon^*_\perp)\Big)\\
&+g^2F_2\Big(\frac{q_+}{u_2^2}(q\epsilon^*_\perp)+\frac{q^2}{u_2^2u_{2-}}(u_2\epsilon^*_\perp)-\frac{q^2}{u_{2-}p_\perp^2}(p\epsilon^*_\perp)\Big)\,.
\end{aligned} \tag{37}$$

5 The diffractive amplitude

The $R\to RRP$ vertex (35) that we obtained has a 4-dimensional form. To reduce it to a transverse vertex we have to study a concrete amplitude that involves this vertex. We choose the simplest amplitude possible: the production of a real gluon in a collision of two quarks, the target quark interacting with the two final reggeized gluons in the colorless state (Fig. 3). This is a simplified part of the diffractive process studied in [16], where the projectile was taken to be a quark loop. Note that there are additional diagrams for this process, which involve two

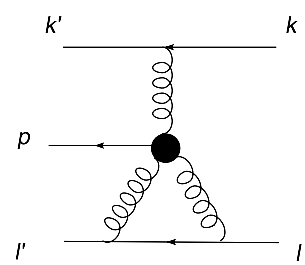

Fig. 3. The diffractive diagram with a $R\to RPP$ vertex

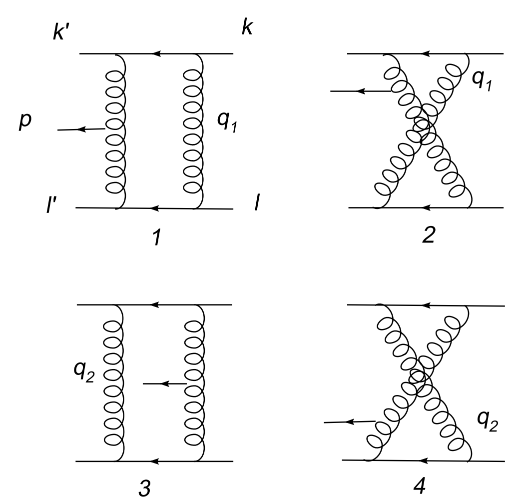

Fig. 4. Diffractive diagrams with two-reggeized-gluon exchange

reggeon exchange between projectile and target with the gluon emitted by the Lipatov vertex (Fig. 4). However, as we shall presently see (and as is known) for them integration over the longitudinal variables in the loop presents no difficulties, so that the whole problem is concentrated in the diagram with the $R \to RRP$ vertex found.

5.1 Two-reggeon exchange

To write the expression for the contribution of two-reggeon exchange (Fig. 4), we have to know the impact factors. For colliding quarks they are trivial. Let the initial momenta of projectile and target be k and l, respectively, with $k_- = l_+ = k_\perp = l_\perp = 0$. Then, say, for the projectile we find the part of the diagram coupled to the reggeons as

$$\frac{1}{(k-q_1)^2 + \mathrm{i}0} u' t^{a_2} \gamma_{\mu_2} (\hat{k} - \hat{q}_1) \gamma_{\mu_1} t^{a_1} u \,. \tag{38}$$

In the BFKL kinematics, as is well known, we can substitute[1]

$$\gamma_\mu \to \frac{\hat{l}}{l_-}\,, \tag{39}$$

where l is the initial momentum of the target. The spin matrix element that results is

$$u' \hat{l} \hat{k} \hat{l} u = 4(kl)^2, \tag{40}$$

where we neglected q_{1+} and q_{2+} as compared with k_+ in our kinematics. With a similar calculation for the target we find the overall factor coupled to the vertex in diagram 1 of Fig. 4 as

$$\frac{16(kl)^2}{(k-q_1)^2 (l+q_1)^2} u' t^{a_2} t^{a_1} u \cdot w' t^{b_2} t^{b_1} w\,, \tag{41}$$

where u, u' and w, w' are the initial and final color wave functions of the projectile and target. In the crossed diagram 2 of Fig. 4, the projectile plus target factor is

$$\frac{16(kl)^2}{(k-q_1)^2 (l'-q_1)^2} u' t^{a_2} t^{a_1} u \cdot w' t^{b_1} t^{b_2} w\,. \tag{42}$$

The impact factors for diagrams 3 and 4 of Fig. 4 have a similar structure with the substitution $q_1 \to -q_2$ and $k, l \leftrightarrow k', l'$.

The overall color factor is obtained by the convolution of color factors from impact factors and the vertex. We also have to take into account the projector onto the colorless state of the reggeons coupled to the target, which is proportional to $\delta_{b_1 b_2}$. Combining these factors, we find the color operator acting on the projectile quark for diagrams 1 and 2 of Fig. 4 to be

$$Q^c = f^{a_2 c a_1} t^{a_2} t^{a_1} \tag{43}$$

and $-Q^c$ for diagrams 3 and 4 of Fig. 4. The color operator acting on the target quark is naturally a unit operator.

Now we pass to the longitudinal integrations in the diagrams of Fig. 4. We start from diagram 1 of Fig. 4 and integrate over q_{1+}. The two denominators are

$$\left[2q_{1-}(q_{1+} - k_+) + q_{1\perp}^2 + \mathrm{i}0\right]\left[2q_{1+}(l_- + q_{1-}) + q_{1\perp}^2 + \mathrm{i}0\right]. \tag{44}$$

A non-zero result is obtained only when the two poles in q_{1+} are on the opposite sides from the real axis. This limits the integration over q_{1-}:

$$-l_- < q_{1-} < 0\,. \tag{45}$$

Denoting the expression for the part of the diagram depending on the longitudinal momenta (the inverse of (44)) by D_1 and taking the residue, we find

$$\int \frac{\mathrm{d}q_{1+}}{2\pi\mathrm{i}} D_1 = \frac{1}{4k_+ q_{1-}^2 + 4k_+ l_- q_{1-} - 2l_- q_{1\perp}^2}\,. \tag{46}$$

Subsequent integration in q_{1-} in the interval $-l_- < q_{1-} < 0$ gives

$$\int_{-l_-}^{0} \mathrm{d}q_{1-} \int \frac{\mathrm{d}q_{1+}}{2\pi\mathrm{i}} D_1 = \frac{1}{2(kl)} \left(\ln \frac{|q_{1\perp}^2|}{2(kl)} + \pi\mathrm{i} \right). \tag{47}$$

Of course, the imaginary part could be immediately found by the Cutkoski rules.

In the crossed diagram 2 of Fig. 4, the second denominator in (44) is changed to

$$-2(q_{1+} - l'_+)(l'_- - q_{1-}) + (l' - q_1)_\perp^2 + \mathrm{i}0\,. \tag{48}$$

Now a non-zero result is obtained for $0 < q_{1-} < l'_- \simeq l_-$. Taking the residue in q_{1+} at the same pole as before, we find

$$\int \frac{\mathrm{d}q_{1+}}{2\pi\mathrm{i}} D_2 = -\frac{1}{4k_+ q_{1-}^2 - 4k_+ l_- q_{1-} + 2l_- q_{1\perp}^2}\,, \tag{49}$$

and the subsequent integration over q_{1-} gives

$$\int_0^{l_-} \mathrm{d}q_{1-} \int \frac{\mathrm{d}q_{1+}}{2\pi\mathrm{i}} D_2 = -\frac{1}{2(kl)} \ln \frac{|q_{1\perp}^2|}{2(kl)}\,. \tag{50}$$

Naturally the crossed diagram contains no imaginary part. In the sum the real parts cancel, and the final result is

$$\int \frac{\mathrm{d}q_{1+}\,\mathrm{d}q_{1-}}{2\pi\mathrm{i}} (D_1 + D_2) = \frac{\pi\mathrm{i}}{2(kl)}\,. \tag{51}$$

Together with the color factor and factors depending on the transverse momenta, this gives the amplitude

$$f^{a_2 c a_1} u' t^{a_2} t^{a_1} u \cdot 4(kl) \frac{1}{q_{1\perp}^2 q_{2\perp}^2} \left(\frac{(q\epsilon)_\perp}{q_\perp^2} - \frac{(p\epsilon)_\perp}{p_\perp^2} \right), \quad q_2 = q - p - q_1\,. \tag{52}$$

This is the standard form for the amplitude described by diagrams 1 and 2 of Fig. 4 in the transverse momentum space (see e.g. [16]).

[1] In this section we use the normalization $ab = a_+ b_- + a_- b_+ + (ab)_\perp$.

Note that the same result is obtained if one passes from the variables q_{1+} and q_{2-} to the energies squared, $s_1=(k-q_1)^2$ and $s_2=(l+q_1)^2$, for the upper and lower quark–reggeon amplitudes in diagram 1 of Fig. 4, and if one deforms the Feynman contours of integration to pass around the unitarity cuts at $s_{1,2}\geq 0$. The contribution from the intermediate quark states at $s_1=s_2=0$ will directly give (50).

Diagrams 3 and 4 of Fig. 4 are calculated in the same manner, and after integration over the longitudinal variables reduce to the standard formulas in terms of the transverse variables.

5.2 Contribution from the $R\to RRP$ vertex

The two terms in the vertex (35) that differ by the permutation $1\leftrightarrow 2$ evidently give the same contribution after integration over the loop momentum and summation over the color indices. So it is sufficient to study only one of them. To use the previous results we choose the second term. The color operator applied to the projectile quark is given by

$$f^{b_2cd}f^{ab_1d}\delta_{b_1b_2}t^a=-N_ct^a\delta_{ac} \qquad (53)$$

and is common for the three terms in the second part of (35).

The first term in this second part, proportional to $(q\epsilon)_\perp$, has the denominators in the direct and crossed terms, which are obtained from (44) and (48) by changing $k\to q$. So the first denominator becomes

$$2q_{1-}(q_{1+}-q_+)+(q-q_1)^2_\perp+\mathrm{i}0\,. \qquad (54)$$

Obviously after integration we shall get the same formulas (47) and (50), in which we have to substitute (ql) instead of (kl) and $(q-q_1)^2_\perp$ instead of $q^2_{1\perp}$. The real parts again cancel, and the result will be given by

$$\int\frac{\mathrm{d}q_{1+}\mathrm{d}q_{1-}}{2\pi\mathrm{i}}(D_1+D_2)=\frac{\pi\mathrm{i}}{2(ql)}\,, \qquad (55)$$

where the terms D refer only to the parts coming from the two propagators in the direct and crossed terms. This has to be multiplied by the color factor and momentum factors coming from the two impact factors. The latter for the target is given by an expression similar to (38) and for the projectile is just $2k_+$. Taking into account the rest of the factors, we finally have the complete contribution from the first term in the second part of (35) as follows:

$$\int\frac{\mathrm{d}q_{1+}\mathrm{d}q_{1-}}{2\pi\mathrm{i}}T_1=4\pi\mathrm{i}(pl)\delta_{ac}u'
t^au\cdot\frac{1}{q^2_{1\perp}q^2_{2\perp}}\frac{q\epsilon_\perp}{q^2_\perp}\,. \qquad (56)$$

This coincides with the first term in the contribution from the Bartels vertex to the diffractive amplitude.

As in the previous calculation of the two-reggeon exchange, the same result is obtained if one similarly introduces the energies squared, $s_1=(q-q_1)^2$ and $s_2=(l+q_1)^2$, for the amplitudes $R+R\to P+R$ and $q+R\to q+R$ and then deforms the Feynman integration contour to close on the unitarity cuts of both amplitudes. The contribution from the intermediate quark + gluon state will immediately give (55).

The situation with the second term is a bit more complicated. In both the direct and crossed term an extra factor $1/q_{1-}$ appears. The integration over q_{1+} is done exactly as before. However, the subsequent integration over q_{1-} cannot be done separately for the direct and crossed term, because of the singularity at $q_{1-}=0$. However, in their sum this singularity cancels and the integration becomes trivial. In fact, after the integration over q_{1+} we find the integral over q_{1-}:

$$\begin{aligned}&\int_{-l_-}^{0}\frac{\mathrm{d}q_{1-}}{q_{1-}}\int\frac{\mathrm{d}q_{1+}}{2\pi\mathrm{i}}D_1\\&=\int_{-l_-}^{0}\frac{\mathrm{d}q_{1-}}{q_{1-}}\frac{1}{4k_+q^2_{1-}+4k_+l_-q_{1-}-2l_-(q-q_1)^2_\perp}\\&=-\int_{0}^{l_-}\frac{\mathrm{d}q_{1-}}{q_{1-}}\frac{1}{4k_+q^2_{1-}-4k_+l_-q_{1-}-2l_-(q-q_1)^2_\perp}\,,\end{aligned} \qquad (57)$$

whereas the corresponding integral of the crossed term is

$$\begin{aligned}&\int_{0}^{l_-}\frac{\mathrm{d}q_{1-}}{q_{1-}}\int\frac{\mathrm{d}q_{1+}}{2\pi\mathrm{i}}D_2\\&=-\int_{0}^{l_-}\frac{\mathrm{d}q_{1-}}{q_{1-}}\frac{1}{4k_+q^2_{1-}-4k_+l_-q_{1-}+2l_-(q-q_1)^2_\perp}\,.\end{aligned} \qquad (58)$$

The sum of (57) and (58) is regular at $q_{1-}=0$. So integration of this sum can be done directly. The resulting real part again vanishes and one finds

$$\int\frac{\mathrm{d}q_{1+}\mathrm{d}q_{1-}}{2\pi\mathrm{i}q_{1-}}(D_1+D_2)=-\frac{\pi\mathrm{i}}{2l_-(q-q_1)^2_\perp}\,. \qquad (59)$$

Adding all the remaining factors, we get for the second term in the second part of (35)

$$\int\frac{\mathrm{d}q_{1+}\mathrm{d}q_{1-}}{2\pi\mathrm{i}}T_2=-4\pi\mathrm{i}(pl)\delta_{ac}u't^au\cdot\frac{1}{q^2_{1\perp}q^2_{2\perp}}\frac{(q-q_1)\epsilon_\perp}{(q-q_1)^2_\perp}\,. \qquad (60)$$

This gives the second part of the contribution corresponding to the Bartels vertex.

Note that if one tries to use here the same simplified method of integration over the energies of the $R+R\to R+P$ and $q+R\to q+P$ amplitudes, closing the contour around the unitarity cuts, then one encounters the singularity at $q_{1-}=0$ with an unknown path of integration around it. If one just neglects this singularity, that is, one takes into account only the unitarity contribution to the discontinuities of the amplitudes, then one gets a result which is twice larger than (59) and so is incorrect.

We are left with the third term in the second part of (35), with a structure that has no counterpart in the Bartels vertex. For the 4-dimensional and 2-dimensional pictures to coincide this contribution has to disappear.

The only denominator in the direct term is

$$(l+q_1)^2 + \mathrm{i}0 = 2q_{1+}(l_- + q_{1\_}) + q_{1\perp}^2 + \mathrm{i}0\,. \tag{61}$$

Obviously the integral over q_{1+} is divergent and can be studied only for the sum of this direct and crossed term. For the latter, the denominator is

$$\left(l'-q_1\right)^2 + \mathrm{i}0 = -2\left(q_{1+} - l'_+\right)\left(l'_- - q_{1-}\right) + \left(l'-q_1\right)_\perp^2 + \mathrm{i}0\,. \tag{62}$$

In the integrand, at large q_{1+} we find the sum

$$\frac{1}{2q_{1+}(l_- + q_{1\_})} - \frac{1}{2q_{1+}\left(l'_- - q_{1\_}\right)}\,. \tag{63}$$

Since $l' = l+q-p$, we have $l'_- - q_{1-} = l_- + q_- - p_- - q_{1-} \simeq l_- - p_- - q_{1-}$ and it is generally not equal to $l_- + q_{1-}$. Therefore, at $q_{1+} \to \infty$ the sum does not generally go to zero faster than $1/q_{1+}$, so that integration over q_{1+} remains divergent even after summing the direct and crossed terms.

The only possibility to give some meaning to this integration is to assume that in the target denominator one may neglect the minus components of all momenta, except for the incoming target momentum l_-. This means that in (61) and (62) we take $l'_- = l_-$ and $q_{1-} = 0$. Then the integration over q_{1+} becomes convergent, and since the poles of the two terms lie at opposite sides from the real axis, it gives a non-zero result, which does not depend on the value of q_{1-}. After that, we find the integral over q_{1-} to be of the form

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}q_{1-}}{q_{1-}}\,. \tag{64}$$

The only possibility to give some meaning to it is to assume that it should be taken as a principal value integral. Then it is equal to zero, and the third term in (35) disappears indeed.

As a result, we find that the longitudinal integration of the 4-dimensional vertex found by the effective action technique requires a certain measure of caution. One finds a piece for which a strict integration is divergent. To overcome this difficulty, one has to neglect the small minus components in the target impact factor (and then do the q_{1+} integration in the trivial manner, closing the contour around the unitarity cut of the reggeon–target amplitude) and afterwards do the remaining q_{1-} integration by the principal value recipe. This result has been found only for the diffractive amplitude. It remains an open question if it has a wider validity and applies also to other cases, which correspond to double and single cuts of the general triple-pomeron amplitude.

6 Conclusions

The Lipatov effective action has demonstrated its advantage for the construction of the vertices for the production of more than one particle from a reggeized gluon, which are necessary for the calculation of next-to-leading corrections to the BFKL pomeron [19]. We have applied the rules of the Lipatov effective action to construct the vertex in which a reggeon passes into two reggeons emitting a gluon. This vertex allows one to study processes in which the number of reggeized gluons changes. As expected, the effective action allows one to build this vertex almost automatically, in contrast to earlier derivations, in which this required a considerable effort. However, the vertex obtained is found in the 4-dimensional form and formally contains singularities in the longitudinal variables.

Its reduction to the pure transverse form meets with some difficulties, related to these singularities. In the diagram with an intermediate gluon, this singularity requires one to do the longitudinal integration, preserving the dependence of the target impact factor on small longitudinal momenta. As a result, the singularity mentioned disappears. In the diagram with a pointlike vertex, on the contrary, small longitudinal momenta are to be neglected in the target impact factor. Then the remaining singularity has to be integrated over by the principal value recipe. After that, one finally obtains the same transverse vertex as found earlier by direct methods.

The fact that one has to apply different rules to treat different contributions certainly leaves a certain feeling of uneasiness. We hope that the only contributions that require special treatment are those with a pointlike emission, which seem to have been dropped from the start in previous derivations using either the dispersion approach or the dominance of large nuclear distances. If this is so, then the use of the effective action preserves its universality except for these exceptional cases. To verify this, one has to study more complicated processes involving the vertex found, such as double scattering off a colorless target. This study is now in progress.

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